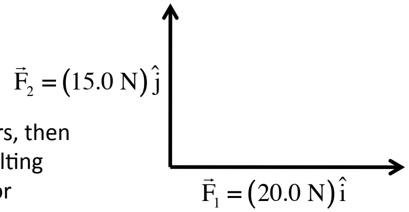
Problem 5.13

The magnitudes of the two forces are 20.0 newtons and 15.0 newtons. If the mass is 5.00 kg, what is the acceleration of the mass if the forces are oriented as shown:

a.) There are two ways to do this. Both are vector manipulations, and both are equally acceptable. I'll show both.



The first way: Add the forces as unit vectors, then divide that sum by the mass "m." The resulting vector will be the acceleration in unit vector notation. That is:

$$\vec{F}_{net} = \vec{F}_2 + \vec{F}_1 = m\vec{a}$$

$$\Rightarrow \quad \vec{a} = \frac{\vec{F}_{net}}{m}$$

$$= \frac{(20.0\ \text{N})\hat{i} + (15.0\ \text{N})\hat{j}}{(5.00\ \text{kg})}$$

$$= (4.00\ \text{m/s}^2)\hat{i} + (3.00\ \text{m/s}^2)\hat{j}$$

Easy check: As this is a 3:4:5 right triangle, we can see by inspection that:

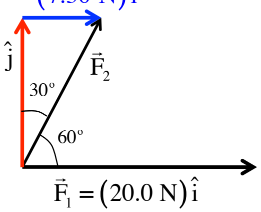
$$(4.00\ \text{m/s}^2)\hat{i} + (3.00\ \text{m/s}^2)\hat{j} = (5.00\ \text{N})\measuredangle 36.9^\circ$$

and our two approaches have given us the same result, just in different notations.

b.) With the vectors rearranged as shown:

$$F_{2,x} = \left[(15.0\ \text{N})\sin 30^\circ\right]\hat{i}$$
$$= (7.50\ \text{N})\hat{i}$$

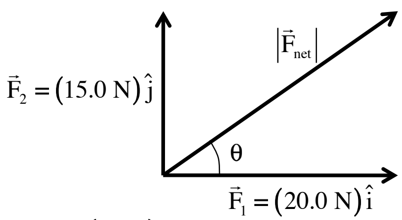
$$F_{2,y} = \left[(15.0\ \text{N})\cos 30^\circ\right]\hat{j}$$
$$= (13.0\ \text{N})\hat{j}$$



The first way: After doing a little manipulation on the sketch and noting that:

$$\vec{F}_2 = F_{2,x}\hat{i} + F_{2,y}\hat{j}$$
$$= (7.50\ \text{N})\hat{i} + (13.0\ \text{N})\hat{j}$$

The second way: Determine the net force in polar notation, then divide by "m." That is:



$$\vec{F}_{net} = |\vec{F}_{net}|\measuredangle \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

$$= \left[(20.0\ \text{N})^2 + (15.0\ \text{N})^2\right]^{1/2}\measuredangle \tan^{-1}\left(\frac{15.0}{20.0}\right)$$

$$= (25.0\ \text{N})\measuredangle 36.9^\circ$$

Sooo:

$$\vec{F}_{net} = m\vec{a}$$

$$\Rightarrow \quad \vec{a} = \frac{\vec{F}_{net}}{m}$$

$$= \frac{(25.0\ \text{N})\measuredangle 36.9^\circ}{(5.00\ \text{kg})}$$

$$= (5.00\ \text{N})\measuredangle 36.9^\circ$$

we can write:

$$\vec{F}_{net} = \vec{F}_2 + \vec{F}_1 = m\vec{a}$$

$$\Rightarrow \quad a = \frac{F_{net}}{m}$$

$$= \frac{(20.0\ \text{N})\hat{i} + \left[(7.50\ \text{N})\hat{i} + (13.0\ \text{N})\hat{j}\right]}{(5.00\ \text{kg})}$$

$$= (5.50\ \text{m/s}^2)\hat{i} + (2.60\ \text{m/s}^2)\hat{j}$$

Doing this the second way would be considerably harder as vectors in polar notation do not add easily (unless you are using graphical vector addition or have right angles—then it's a snap). As such, we won't try it. We can calculate our acceleration in polar notation, though. It is:

$$(5.50\ \text{m/s}^2)\hat{i} + (2.60\ \text{m/s}^2)\hat{j} = \left[(5.50\ \text{m/s}^2)^2 + (2.60\ \text{m/s}^2)^2\right]^{1/2}\measuredangle \tan^{-1}\left(\frac{2.60}{5.50}\right)$$

$$\Rightarrow \quad \vec{a} = (6.08\ \text{m/s}^2)\measuredangle 25.3^\circ$$